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TWO-DIMENSIONAL APERTURE PHOTOMETRY: SIGNAL-TO-NOISE RATIO OF POINT-SOURCE OBSERVATIONS AND OPTIMAL DATA-EXTRACTION TECHNIQUES

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ABSTRACT

New methodology and associated software techniques for use when performing two-dimensional aperture photometry with data from areal detectors are presented. It is shown that applying simple Poisson statistics to point sources only works in certain limited regimes and that for many situations the "CCD equation" must be employed to get accurate results. Sky or background subtraction is again found to be of critical importance and, of five background estimation methods tried, each had some difficulties in producing a "correct" background value. CCD growth curves are introduced as a means of correcting the flux from faint or crowded sources, when measured with very small (optimum) apertures. These optimum apertures are of interest as they are shown to give large increases in S/N for point-source observations. The usage of these small apertures along with CCD growth curves is discussed and presented with emphasis on their use for time-resolved CCD photometry.

Key words: areal detectors–data analysis–photometry

"There are two kinds of truths; those of *reasoning* and those of *fact*.
Truths of reasoning are necessary, and their opposite is impossible;
And those of fact are contingent, and their opposite is possible."

Gottfried Wilhelm Von Leibniz
—*Monadology* (1714)

1. Introduction

The basic software to perform two-dimensional digital aperture photometry has been available for some time within software packages such as the KPNO MPC (Adams *et al.* 1980) and DAOPHOT (Stetson 1987). Recently, due to the decreased support for conventional photoelectric photometry and increased availability of CCDs and other 2-D detectors, the art of 2-D aperture photometry from digital detectors has come under close scrutiny. Walker (1984) has investigated the accuracies of observations of bright photometric standards using CCDs. Howell and Jacoby (1986) looked at two methods of using CCDs as time-resolved photometers, and later Howell, Mitchell, and Warnock (1988) discussed the essential procedure for CCD differential photometry. The NOAO newsletters over the past few years have discussed some of the intricacies of their CCDs and CCD properties in general while Djorgovski (1984) and Gilliland and Brown (1988) have looked critically at some typical reduction steps used as "black boxes" within the common software reduction packages available. Other items, such as stellar profiles on pixel arrays (King 1971, 1983) and A/D conversion techniques in relation to digital photometry (Opal 1988), are also becoming prevalent. For a general review of the detailed workings of CCDs for astronomy, a recent review article by Mackay (1986) is highly recommended.

When performing 2-D aperture photometry, it is important to calculate the correct signal-to-noise ratio (S/N) for each point-source observation and to understand how to determine the best software aperture to use when extracting the data. I present below some results which provide new insight into these techniques. Some of these results are based on ideas developed through discussions with colleagues and through hearing of others' speculations. For these I am thankful.

Section 2 discusses the S/N of a point source and when one needs to abandon simple photon statistics in order to determine the accuracy of the data. In Section 3 sky (background) determination and growth curves of stellar images on CCDs are discussed. A possible alternative to profile fitting is proposed. Section 4 deals with S/N as a function of (radial) distance from the point-source center and how this can be used to alleviate some of the problems of Section 3 as well as to improve overall error.

2. S/N of an Observation of a Point Source

Poisson (photon) statistics are usually invoked when dealing with photon-counting devices with the S/N being given by $\sqrt{N}$, where N is the total photon count from the source. For cases where the system and background noise contributions are negligible, $\sqrt{N}$ gives the correct answer since the only noise is from the photon arrival rate itself. However, if other noise sources are significant, they must be addressed in the error determination. For a CCD, the S/N is given by the well-known "CCD equation" which has the form

$$S/N = \frac{N_*}{\sqrt{N_* + n_{pix}\,(N_s + N_d + N_r^2)}} \; . \qquad (1)$$

The individual terms are: N_*, the total (sky subtracted) number of photons from the source; n_{pix}, the number of pixels contained within the software aperture; N_s, the total number of sky photons per pixel; N_d, the dark current in photons per pixel; and N_r, the read noise in electrons per pixel. This equation will only equal $\sqrt{N}$ if the extra terms in the denominator are acceptably small compared with N_*. For most modern astronomical CCD systems the dark current can safely be ignored. The sky value is variable and greatly affected by such things as the lunar phase, exposure time, and general sky conditions. Read noise has slowly approached the 1–10 electron per pixel range, but many systems still in operation have this term as the dominant noise source in some regimes.

In order to investigate when the extra terms in equation (1) can be ignored, simulations of performing aperture photometry on point sources on CCDs have been done. The expected S/N from equation (1) was then compared with what would be calculated from a simple $\sqrt{N}$ model (see Fig. 1). The simulations were for an integration time of 300 seconds and read noises of 79 (high) and 5 (low) electrons per pixel. These are roughly equal to a typical RCA CCD and a TEK CCD, respectively. The number of expected star and (dark) sky photons were taken from the KPNO CCD manuals for the 0.9-m and 4-m telescopes. The general shape of the curves is similar, but the instrumental magnitude at which the simple $\sqrt{N}$ approximation fails varies by many magnitudes. For example, in Figure 1(c), Poisson statistic calculations would give a 20% error for an 18th-magnitude star observed under the conditions stated above. There are, of course, many other cases (e.g., different integration time, different sky background, etc.), but these examples should be enough to alert the reader. Figure 1 does show, however, that for obtaining a quick reference magnitude from a *bright* photometric standard, it is pretty safe to use $\sqrt{N}$ as a measure of the error.

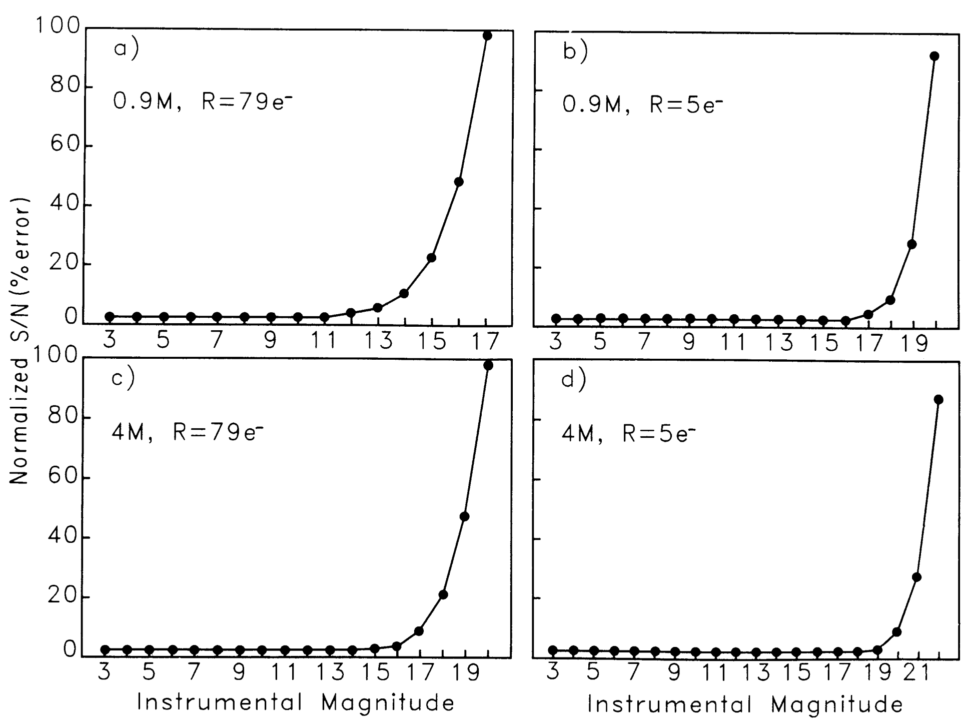

FIG. 1–The plots show the percent error which occurs if simple Poisson statistics are used instead of the formal CCD equation when calculating the expected S/N for a point-source observation. (a) and (b) show the results for the 0.9-m telescope while (c) and (d) show the results for the 4-m telescope. The integration time in all cases is 300 seconds. The instrumental magnitudes are roughly equal to the real magnitudes expected in V for dark skies at Kitt Peak. The data used to generate the curves were taken from the KPNO CCD manuals. The two choices of read noise, 79 and 5 electrons, are typical of RCA and TEK CCDs, respectively. Note that the horizontal scale is not the same in all the plots and the initial (horizontal) y-value in each plot = 1.0.

3. Sky (Background) Determination and CCD Growth Curves

For two-dimensional detectors, the standard method for sky or background determination has been to take an annulus around the source or an area offset from the source, look at the pixel values within this area, and use some algorithm to determine the value that is to be assigned to the background. This value is then subtracted on a per-pixel basis, from the total counts within the source, to yield a measure of the collected flux. For bright sources this can be easy to do as either the background is zero (due to short integrations) or is very low and the counts from the source are generally much larger. When we look at fainter sources, however (e.g., sky limited measurements), the story is different. For example, a point source has a very large area (the stellar wings), where the object flux and the background (sky) flux compete for dominance within a given pixel. This is even more so for non-point-source images because of the very extended areas over which background and other noise sources are very important. Sky determination is therefore very critical to obtaining accurate measurements; this can be expressed as "the sky's the limit!" (Jacoby 1988, private communication).

In order to better understand background subtraction, the flux variation as a function of distance from the object center has been looked at for point sources on various CCDs. Different background determination schemes have been used, all of which were relatively simple algorithms, as interest in low CPU usage methods was desired for reasons related to other aspects of this project. As the flux is added up within increasingly larger radial apertures, it is expected that all the flux will be included eventually, and in going to larger radii the total flux from the point source will not increase. This is not the case if one does the sky subtraction incorrectly. Figure 2[1] shows the results for three point sources, two of which are not sky limited (peak counts above sky of ~220 analog-to-digital units (ADU)), while the third is one of the faintest objects discernible on the frame (peak count above sky ~70 ADU). The magnitudes given are instrumental. Adding ~5.5 to the magnitude values given roughly yields real magnitudes for these stars that would be obtained with a 36-inch (91-cm) telescope using a typical RCA CCD system, a 300-sec integration, and a dark sky.

The figure shows that, for the two brighter sources, a total flux sum is eventually reached, so the background subtraction is fairly good. The sky-limited object behaves very differently. The sky subtraction is not done well at all and, as one goes to larger radii, more sky-dominated pixels are included and the apparent flux from the source decreases. The method used to estimate the sky gave

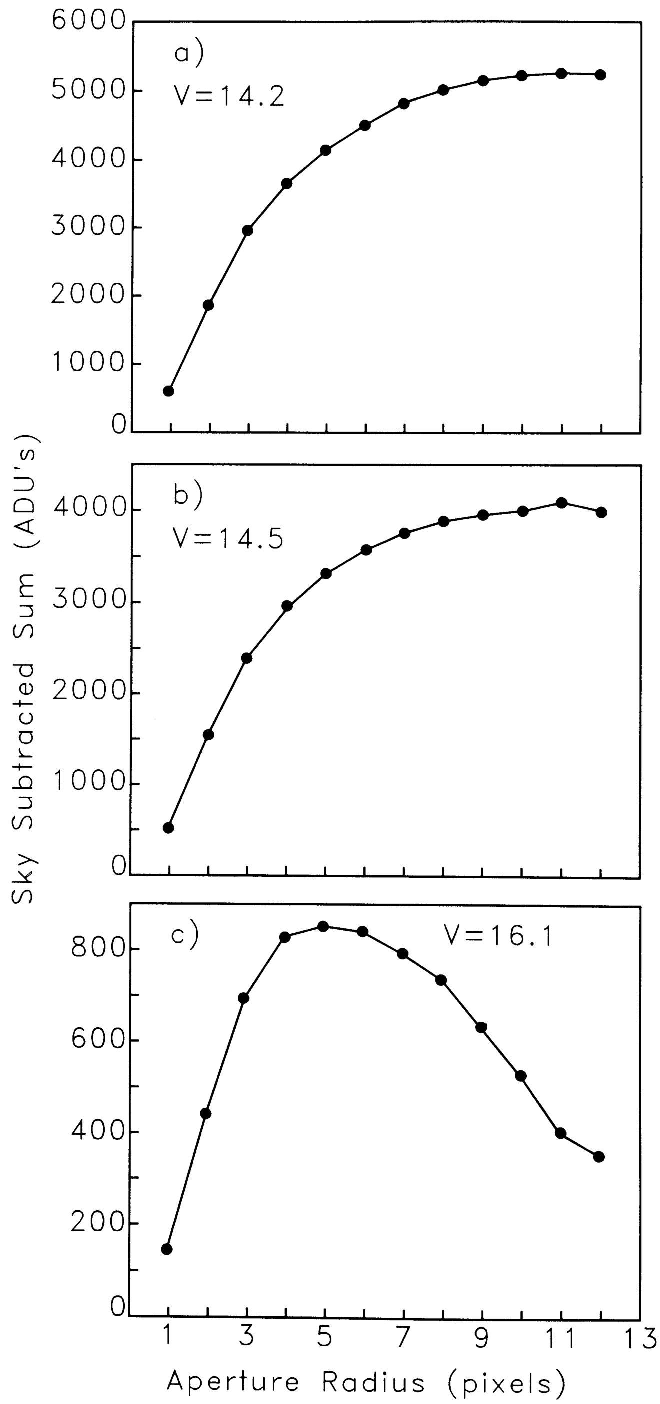

FIG. 2–Sky subtracted count as a function of radius for three point sources, one of which is sky limited, are shown. The background was determined by use of a simple average. The image scale is 0.4 arc sec per pixel.

[1]The data presented in Figures 2–6 are from a frontside-illuminated Tektronics 512 × 512 CCD which was used on a 1-m telescope. The read noise was 7.5 e^-/pixel and a gain of 14.5 e^-/ADU was used. The sky level per pixel per integration was about 130 ADUs and was determined from an annulus around each star of inner and outer radii of 12 and 16 pixels, respectively. There were no apparent chip defects or faint companions seen in any of the measurements. The figures represent typical results that have been obtained from similar measurements made with TI and RCA CCDs as well.

slightly too large a value, and subtracting this value over increasingly larger areas decreases the net flux from the source. Note that the fainter source flux peaks around a radius of 5 pixels. Figure 3 contains the same three sources with their instrumental magnitude plotted. It shows that if a constant aperture is applied to extract all three objects, very large errors in the determined magnitude of sources as a function of their faintness could occur.

Figures 2 and 3 were constructed using a simple average background determination from an annulus surrounding each object. In an attempt to find better background determination methods, a median, a weighted mode, Lucy smoothing (Lucy 1974), and a weighted histogram centroid were also tried. Most of the algorithms worked equally well as in the case of Figures 2 and 3, i.e., the brighter sources were well determined and the fainter object contained large deviations. The weighted mode actually fit the faint source fairly well but gave background values too small for the bright sources which caused an apparent increase in their flux as larger radii were used (see Fig. 4). A point of interest here is that the background value determined from the two methods (Figs. 2 and 3 compared to that for Fig. 4) differed by only about 1 ADU! Even these small differences are important for faint sources. As the measuring aperture increases the number of "pure" sky pixels contained within it rises, and they quickly dominate over those of star + sky. Therefore, when the overestimated (underestimated) sky value is subtracted, the contribution from the star is decreased (seemingly increased) leading to an apparent decrease (increase) in flux. The impact of truncation errors during acquisition and processing can, therefore, be potentially enormous.

It might appear, therefore, that individual radii and individual background determination techniques for each object on a given frame are necessary. This is very costly both in terms of the necessary software flexibility and the CPU time needed to perform the calculations. User interaction would also probably be needed until a smart set of algorithms were written that would make intelligent choices from results similar to those plotted in Figures 2–4.

A possible solution to this difficulty is to construct growth curves for objects on each CCD frame or set of frames. Using a few bright point sources on a CCD frame, one can make plots similar to those shown in Figure 5. Five sources from a CCD frame have been used to show how each source "grows" as one uses larger and larger radii in which to add up the flux. If one assumes that ALL point sources on a given frame should follow a similar growth curve but that some do not due to incorrect background subtraction, crowded fields, variable background behind the source, or other errors that affect fainter objects more than brighter ones, then one can use these bright sources to find the correct growth curve. The sources themselves determine the shape of their growth curve; thus, this method is a kind of azimuthal-averaged, profile-fitting technique.

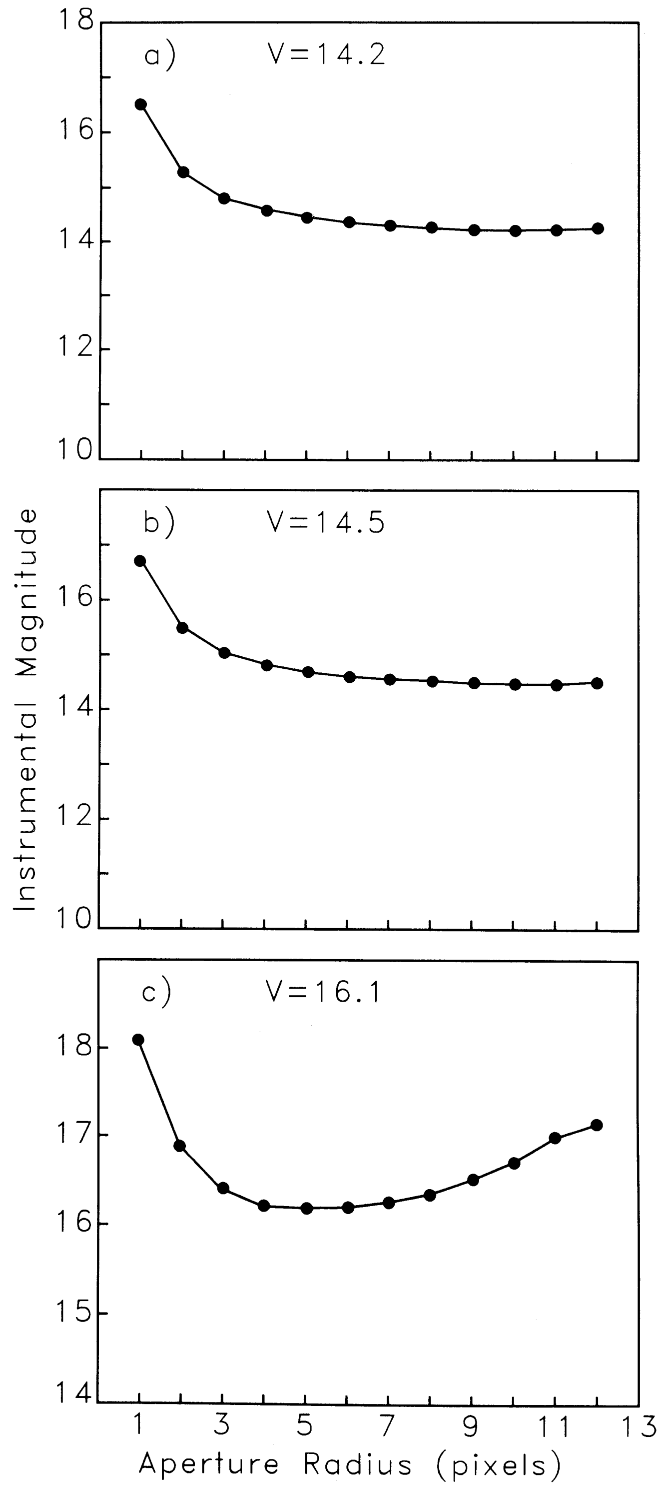

FIG. 3–The same as Figure 2 except that instrumental magnitude is plotted. Note the error that can occur depending on the choice of extraction aperture size. The image scale is the same as in Figure 2.

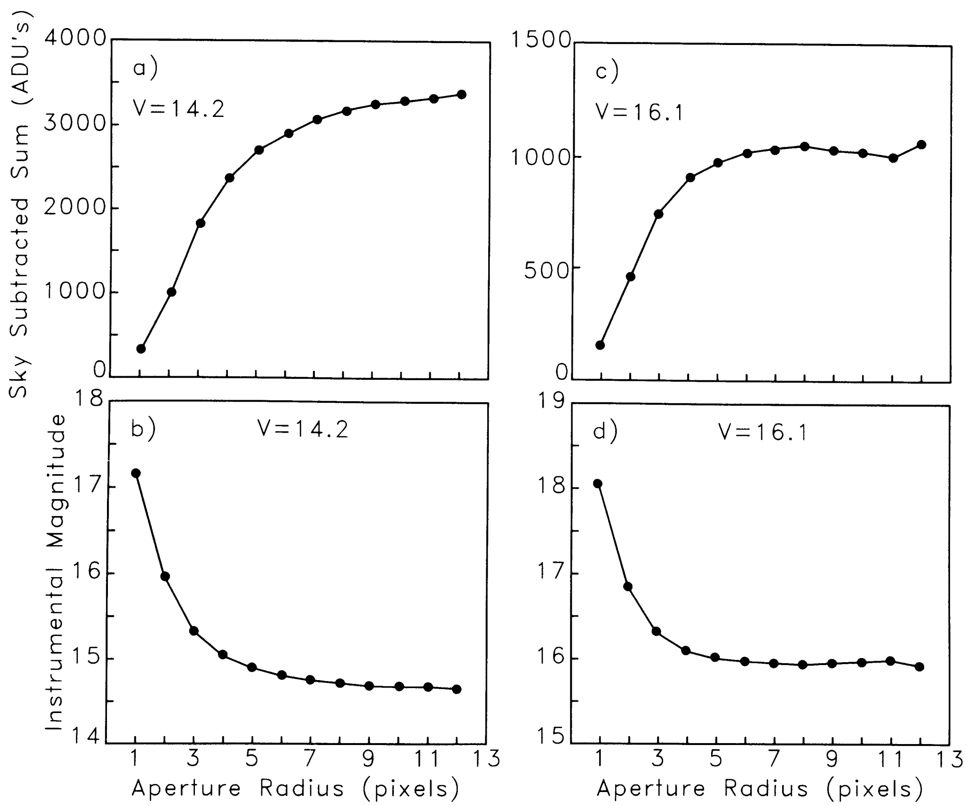

FIG. 4–Same as Figures 2 and 3 (for the brightest and faintest source), except a weighted mode was used when determining the sky background. Note how the bright star is not fit very well in this case and its flux continues to increase. The image scale is the same as in Figure 2.

Fitting an average curve to that defined by the bright sources (e.g., out to a radius of 10 or 12 pixels for the top three in Fig. 5) gives a model profile which can be used to scale measurements made of faint or imperfect sources. These sources can be extracted with small apertures (see next section) and their "true" total flux determined by scaling via the constructed curve. An estimate of the error in this procedure is the spread of the individual curves used for the fit. At a radius of 10 pixels, for example, where it appears that all the flux has been counted, the error of the fit is ~0.05 magnitude. Compare this with the error one would get by using the curve of Figure 3(c) which is approximately 0.5 magnitude at a radius of 10 pixels. The error within a growth curve is discussed in the next section and is found to be a minimum at some small radial value and increases along the curve in both directions. A better fit might be obtained by using some weighting function determined by the S/N at each radius.

4. S/N as a Function of Radii and Its Use In Aperture Photometry

An examination of equation (1) shows that as the noise terms increase for a given N_* the S/N goes down, i.e., the precision of the measurement worsens. If the S/N as a function of apertures with increasing radii is calculated, curves, as shown in Figure 6, are produced.

Three important results should be observed in this figure. The S/N is a maximum at fairly small radii (approximately the FWHM of the stellar profile but not in general equal to it) and decreases in both directions away from that point, the maximum S/N is *not* necessarily at the same radius for all sources, and the decrease in S/N away from the maximum is dramatic for objects of all flux levels. Many such sets of data have been constructed and in all cases these three results have been observed. Figure 6b shows how the photometric precision behaves as a function of radius. These plots show that the data used for each aperture have a unique error associated with them. Thus, the error for each growth-curve datum is not a constant along the curve but is a function of radius and governed by equation (1).

The best S/N is thus obtained by extracting the source flux from a relatively small aperture. The decrease in S/N as one moves away from this maximum is due to too few source pixels for smaller radii and too many "noisy" pixels for larger radii. This technique is quite an improvement over the usual method of picking an aperture that is slightly larger than one that appears to include all the light from the source. For example, for the objects in Figure 6, an aperture visually picked from a radial profile plot on the basis of including all the light would be about 8 pixels in radius and give up to a 60% decrease in photometric

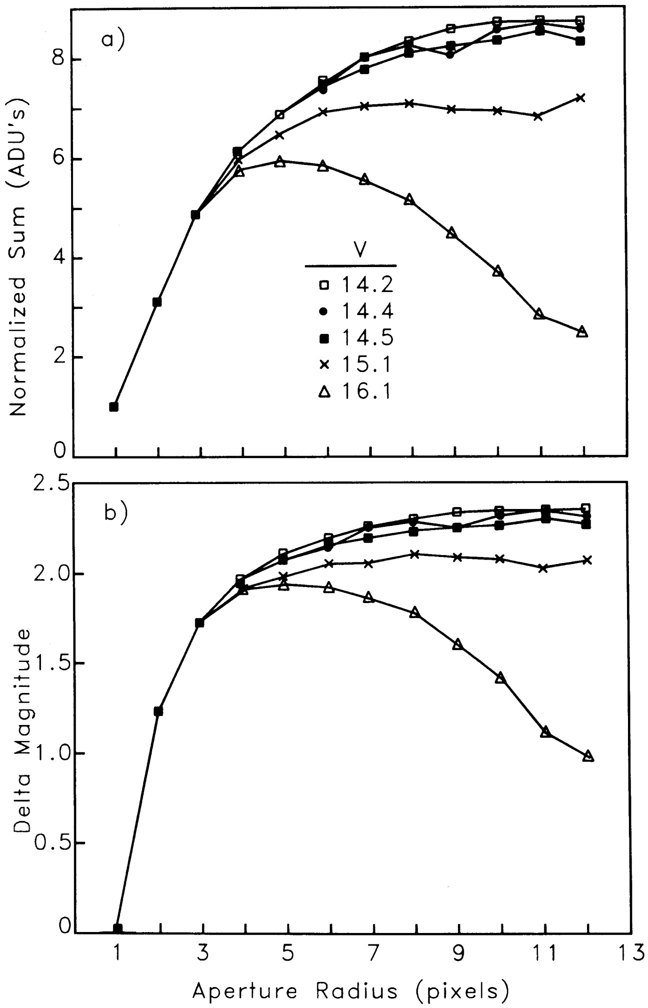

FIG. 5–(a) shows typical CCD growth curves of point sources for five sources from a single CCD frame. A median filter was used for background determination. Note how the top three curves form a fairly tight relation and, as the source faintness increases, the curves diverge greatly from the mean. (b) shows the curves plotted as magnitude differences from their individual values at a radius of one pixel. The image scale is the same as in Figure 2.

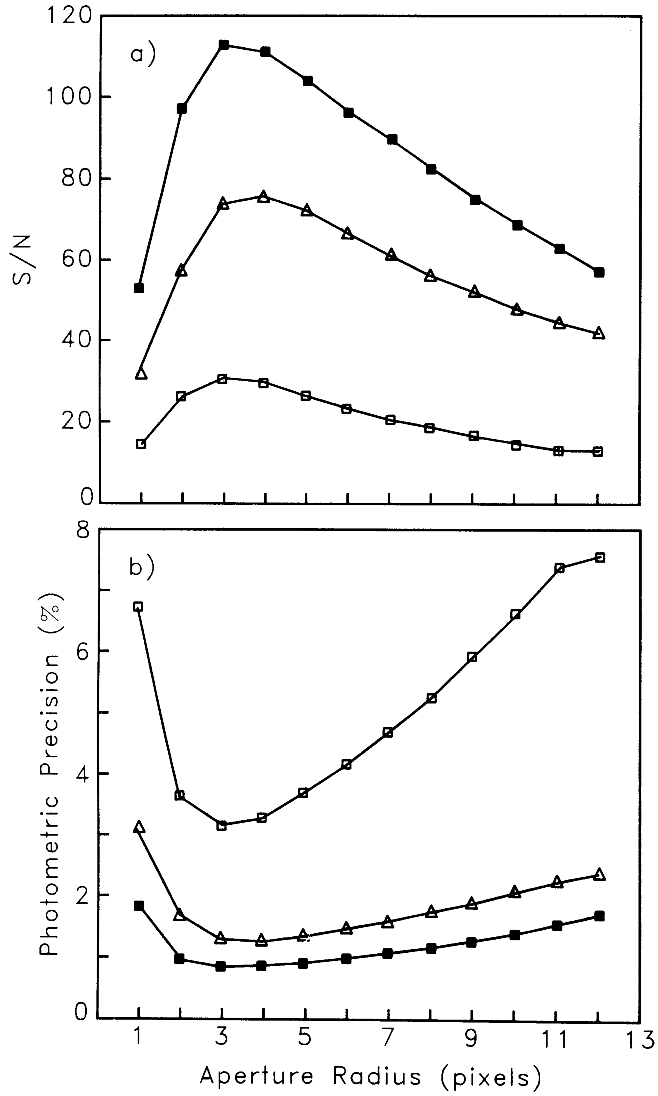

FIG. 6–S/N and photometric precision are plotted as functions of aperture radius for the same three point sources as before. The plots show that for a specific radius, which is fairly small, a point source has a maximum S/N and photometric precision. This maximum is not necessarily at the same radius for different objects. The symbols are ■; $V = 14.2$, △; $V = 14.5$, □; $V = 16.1$. The image scale is the same as in Figure 2.

precision for the sources considered here.

An obvious flaw with the above method of maximum S/N extraction is that one does not include all the flux from a source. Light contained within the wings lies outside the small radius used. Using the growth curves discussed earlier, the amount of "missing" flux that should be added to that measured within the small radius can be determined, yielding the total source flux. In this manner, measurements of a source use only those pixels where the S/N is high and do not use the large area of pixels (i.e., the wings), for which the sky and other noise sources become dominant. This is akin to 2-D profile fitting but is computationally easier and gives similar results.

Variability studies in photometric weather or when using the technique of differential CCD photometry (see Howell *et al.* 1988) are important applications of the above analysis. Generally, for a given observation, only flux differences as a function of time are needed, not the total flux. It has been shown by Howell and Szkody (1988) that very faint sources ($V \lesssim 21$) can be monitored for variability with 1-m telescopes and time resolutions of a few minutes quite precisely (e.g., ~0.005 mag). Using optimum extraction radii will now allow these measurements to be made with even shorter time resolutions and/or for fainter objects.

Another major problem that is overcome by using optimum aperture size is that the method of background determination becomes much less critical to the entire data-extraction procedure. This is because within the small radius (i.e., small area) used, small differences (errors?) resulting from the various methods for background determination are relatively unimportant to the final outcome of the 2-D photometry. Therefore, any of the methods for sky determination discussed in Section 3 work equally well if an optimum aperture is used. Even when not using optimum apertures, it is clear that for faint sources, as few as possible "pure" sky pixels within the measuring aperture are desired. The method used (e.g., simple averaging) can be the one most suited to the observational and computational needs of the situation (e.g., real-time reduction). The other sources of noise included in equation (1) also carry much less weight within the small area used, as source photons dominate in each pixel.

5. Summary

The method of aperture photometry with two-dimensional data will undoubtedly increase in the future. This will be due to the general rise in the availability of 2-D detectors both in the visible and other wavelength regions; the decrease in the support for PMTs; growing user awareness of the properties, uses, and potential of 2-D detectors; and, hopefully, continued advances in data-handling procedures.

The methods presented here have greatly improved on the "standard" aperture photometry methodology while still being computationally easy to implement. This should be of interest to users who want to reduce their 2-D data in real time, have limited computer resources (e.g., memory, CPU speed), or are performing differential analysis yet want to keep the accuracy and other benefits of true 2-D profile fitting.

By using the methods outlined here for 2-D aperture photometry data extraction and analysis, it has been shown that increased accuracy and/or time resolution occurs. These increases can be quite dramatic depending on the exact nature of the data. This paper does not mean to suggest that the "standard" methods employed are incorrect, only that improvements can be made. Work on this subject is continuing, with CCD growth-curve methodology, usage, and error properties being of prime interest.

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